

# ACCURATE DESIGN OF MICROSTRIP DIRECTIONAL COUPLERS WITH CAPACITIVE COMPENSATION

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## ABSTRACT

This paper presents an accurate design of microstrip directional couplers with high directivity using capacitive compensation. The method utilizes symmetry analysis and equivalency principals to develop closed form solutions of the compensating capacitance and a new odd mode characteristic impedance necessary to realize an ideal microstrip directional coupler. The design approach is valid for any degree of coupling thereby overcoming limitations of previous approaches to this design concept.

## I INTRODUCTION

Quadrature directional couplers consisting of parallel-coupled microstrip transmission lines are used extensively because they are easily incorporated in microwave and millimeter wave integrated hybrid monolithic circuits.

Microstrip transmission line has an inhomogeneous dielectric—partly dielectric substrate, partly air. For this reason, odd and even mode phase velocities are unequal. This inequality manifests itself in the couplers poor directivity. The directivity performance becomes worse as the coupling is decreased, or as the dielectric permittivity is increased [1].

There are several methods of improving the directivity of such couplers, i.e., serrat the gap between the conductors [2], adding lumped capacitors at each end of the coupler [3] and selecting two or more different permittivities and their thicknesses for the multi-level substrate [1].

The second method has been analyzed by Schaller [3] and Kajfez [4]; however, the analysis is only approximate. The developed equations for the capacitance determination are nearly true for tight coupling; however, the center frequency prediction is lower than desired. This necessitates foreshortening of the coupled section. Further, for loosely coupled sections, the equations are no longer valid.

This paper addresses the development of design equations for the capacitively compensated directional coupler with ideal directivity using symmetry and equivalency techniques. The approach generates closed form solutions for the compensating capacitance and a new odd mode characteristic impedance necessary to realize an ideal microstrip directional coupler. The results provide accurate designs of quadrature microstrip directional couplers valid for tight and loosely coupled sections.

## II METHOD OF ANALYSIS

The capacitively compensated microstrip directional coupler consists of two symmetrical inner conductors with a ground plane of length  $l$ , separated by a dielectric substrate with relative dielectric constant  $\epsilon_r$ . At the edges of the coupled section, there are two lumped capacitors as shown in Figure 1.

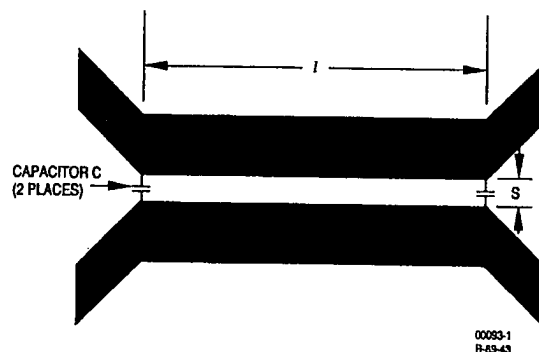


FIGURE 1. Capacitively Compensated Microstrip Directional Coupler

The method of analysis makes use of the physical symmetry of this directional coupler. By applying symmetric (even mode) and antisymmetric (odd mode) excitation to two collinear ports of the directional coupler, the four-port problem is reduced to that of solving two, two-port problems. The two, two-ports to be analyzed are shown schematically in Figure 2. The coupled region is characterized by two transmission lines with characteristic impedances,  $Z_{0o}$  and  $Z_{0e}$ , and two effective relative dielectric constants,  $\epsilon_{effo}$  and  $\epsilon_{effe}$ . The second subscript refers to odd and even modes respectively. Notice that the compensating capacitors do not affect the even mode representation.

The standard procedure is to describe these two circuits using the ABCD matrix notation as:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}_e = \begin{pmatrix} \cos \theta_e & jZ_{0e} \sin \theta_e \\ jY_{0e} \sin \theta_e & \cos \theta_e \end{pmatrix} \quad (1)$$

and

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}_o = \begin{pmatrix} \cos \theta_o - Z_{0o} 2\omega C \sin \theta_o & jZ_{0o} \sin \theta_o \\ j \left( 4\omega C \cos \theta_o + (Y_{0o} - Z_{0o} (2\omega C)^2) \sin \theta_o \right) & \cos \theta_o - 2\omega C Z_{0o} \sin \theta_o \end{pmatrix} \quad (2)$$

Then, these matrices are used in developing the overall scattering matrix for the directional coupler.

Directivity, the performance characteristic of concern, is defined as the difference between isolation and coupling expressed in dB. Both of these characteristics are deduced from the scattering matrix of the directional coupler, i.e.

$$\text{Coupling} = 20 \log_{10} \left( \frac{1}{|S_{21}|} \right) \quad (4)$$

Directivity (D), is then given by

$$D = 20 \log_{10} \left( \frac{|S_{12}|}{|S_{13}|} \right) \quad (5)$$

$$\text{Isolation} = 20 \log_{10} \left( \frac{1}{|S_{13}|} \right) \quad (3)$$

In this paper, a very important deviation is introduced; that is, to find an equivalency between the actual realization and the ideal odd mode representation. This description is depicted in Figure 3.

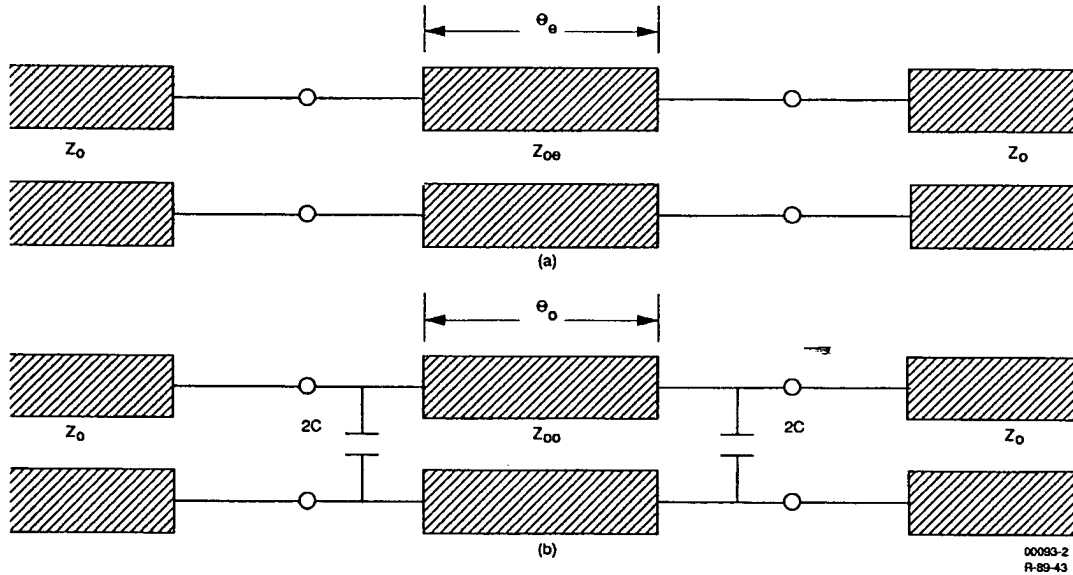


FIGURE 2. Directional Coupler with Capacitive Compensation  
a) Even Mode Equivalent Circuit  
b) Odd Mode Equivalent Circuit

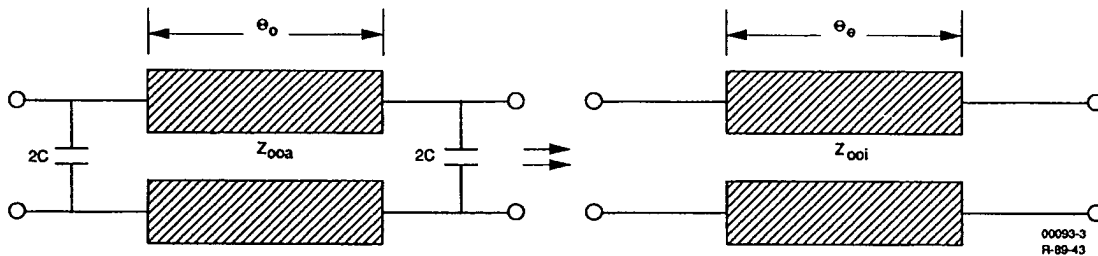


FIGURE 3. Equivalence Between Actual And Ideal Odd Mode Representation

where  $Z_{0oa}$  = actual odd mode characteristic impedance  
 $Z_{0oi}$  = ideal odd mode characteristic impedance

$\theta_o$  = actual odd mode electrical length of the coupled section  
 $\theta_e$  = even mode electrical length of the coupled section

Observe that the ideal odd mode electrical length is made equal to the even mode electrical length. Further, the actual characteristic impedance of the odd mode is different from the ideal. The ABCD matrix representation is chosen to find the conditions that will satisfy this equality. Specifically, each entry of the ABCD matrix network are equated and conditions determined. The matrix description for the actual odd mode representation are given by Equation (2) and for the ideal representation by

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}_i = \begin{pmatrix} \cos \theta_o & jZ_{ooi} \sin \theta_o \\ jY_{ooi} \sin \theta_o & \cos \theta_o \end{pmatrix} \quad (6)$$

Equating the A and B element entries in Equations (2) and (6), we get

$$\cos \theta_o = \cos \theta_o - 2\omega C Z_{oo} \sin \theta_o \quad (7)$$

$$Z_{oo} \sin \theta_o = Z_{ooi} \sin \theta_o \quad (8)$$

Combining (7) and (8)

$$\cos \theta_o = \cos \theta_o - 2\omega C Z_{ooi} \sin \theta_o$$

and recognizing that at the center frequency

$$\theta_o = \pi/2 \quad (9)$$

then

$$C = \frac{\cos \theta_o}{2\omega C Z_{ooi}} \quad (10)$$

Since

$$\theta_o = \frac{2\pi}{\lambda_{go}} l = \frac{\pi}{2} \sqrt{\frac{\epsilon_{effo}}{\epsilon_{effe}}} \quad (11)$$

The compensating capacitance dependency becomes

$$C = \cos \left( \frac{\pi}{2} \sqrt{\frac{\epsilon_{effo}}{\epsilon_{effe}}} \right) \frac{1}{2Z_{ooi}\omega} \quad (12)$$

The actual odd mode characteristic impedance is deduced to be

$$Z_{ooa} = \frac{Z_{ooi}}{\sqrt{1 - \left( \cos \left( \frac{\pi}{2} \sqrt{\frac{\epsilon_{effo}}{\epsilon_{effe}}} \right) \right)^2}} \quad (13)$$

Equation (7) demands that  $Z_{ooa}$  be higher than  $Z_{ooi}$ . To achieve this, the inner-conductor must become narrower and the separation must increase to keep the even mode characteristic impedance constant. This of course, is possible, as the example undertaken later will demonstrate.

With this equalization, at the center frequency at which the directional coupler is designed to operate, we find that Equations (1) and (2) reduce to:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}_e = \begin{pmatrix} 0 & jZ_{oe} \\ jY_{oe} & 0 \end{pmatrix} \quad (14)$$

and

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}_o = \begin{pmatrix} 0 & jZ_{ooi} \\ jY_{ooi} & 0 \end{pmatrix} \quad (15)$$

This is identical to an ideal directional coupler with the following S-parameters.

$$(S) = \begin{pmatrix} 0 & k & -j\sqrt{1-k^2} & 0 \\ k & 0 & 0 & -j\sqrt{1-k^2} \\ -j\sqrt{1-k^2} & 0 & 0 & k \\ 0 & -j\sqrt{1-k^2} & k & 0 \end{pmatrix} \quad (16)$$

$$k = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} \quad (17)$$

$$Z_o = \sqrt{Z_{oe} Z_{oo}} \quad (18)$$

The fact that the isolation (S14) is zero, leads to infinite directivity. Of course, it is understood that this ideal directivity will degrade when losses are introduced and frequency departs from design center.

### III. APPLICATION OF THEORY

The use of the developed formulas will be demonstrated via a design example of an edge coupled microstrip directional coupler. The substrate properties to be used in this example are shown in Table 1.

TABLE 1. Substrate Properties for Microstrip Transmission Line Application

PARAMETERS	VALUES
$\epsilon_r$	9.6
$h$	635 $\mu\text{m}$
$t$	3 $\mu\text{m}$

Table 2 shows the pertinent information regarding the microstrip directional coupler requirements and coupled line realizations for both uncompensated and compensated structures. EEsol's LineCalc software was used to generate the coupled line structure characteristics.

TABLE 2. Properties of Coupled Line Uncompensated and Compensated Microstrip Directional Coupler

PARAMETERS	UNCOMPENSATED	COMPENSATED
Center Frequency	12.0 GHz	12.0 GHz
Coupling (dB)		
Desired	-7.0 dB	-7.0 dB
Achievable	-7.25 dB	-7.0 dB
$Z_o$	50 ohms	50 ohm
$Z_{oe}$	80.85 ohms	80.85 ohms
$Z_{oo}$	30.92 ohms	31.71 ohms
$\epsilon_{effe}$	7.20	7.20
$\epsilon_{effo}$	5.31	5.32
$W$	443.7 $\mu\text{m}$	440.2 $\mu\text{m}$
$S$	80.8 $\mu\text{m}$	89.4 $\mu\text{m}$
$L$	2504.9 $\mu\text{m}$	2326.9 $\mu\text{m}$
Directivity		
Desired	Infinite	Infinite
Achievable	13.25 dB	Infinite

Using the uncompensated results of Table 2, Equations (12) and (13) and calculating the compensating capacitance, the actual odd mode characteristic impedance and the length of the coupled section based on the even mode velocity of propagation can be derived. The results are:

$$C = \frac{1}{4\pi \times 12 \times 10^9 \times 30.92} \cos\left(\frac{\pi}{2} \sqrt{\frac{5.31}{7.2}}\right) = .047 \text{ pf} \quad (19)$$

$$Z_{0oa} = \frac{30.92}{\sqrt{1 - \left(\cos\left(\frac{\pi}{2} \sqrt{\frac{5.31}{7.2}}\right)\right)^2}} = 31.71 \text{ ohms} \quad (20)$$

$$l = \frac{3 \times 10^5}{4 \times 12 \sqrt{7.2}} = 2329.2 \mu\text{m} \quad (21)$$

To achieve the same even and different odd mode characteristic impedance, compared to the uncompensated case, it was necessary to reduce the inner conductor width and increase the separation between conductors. These modifications cause third order changes in the effective dielectric constant of the coupled structure. If these changes were more significant, it would be necessary to go through another iteration of the design cycle.

Scrutiny of results indicates ideal directivity for on-frequency operation, and no change in coupling value.

#### IV. CONCLUSIONS

Microstrip directional couplers suffer from poor directivity because of inhomogeneous dielectric, i.e., partly dielectric substrate, partly air. For this reason, odd and even modes excited in the coupled region exhibit different velocities and consequently different wavelengths. It is possible to compensate for this inequality by introducing lumped capacitors at the edges of the coupled region. Attempts at accurate theoretical design of these couplers, for any degree of coupling, have not been very successful.

This paper fills the void, by presenting an accurate approach to the design of microstrip directional couplers with high directivity using capacitive compensation. The method is valid for tight and loosely coupled structures and overcomes all limitations of previous approaches to this design concept. The method is validated via a design example.

#### V. REFERENCES

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